MATH 54 - FINAL EXAM STUDY GUIDE

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This is the study guide for the final exam! It says what it does: to guide you with your studying for the exam! The terms in **boldface** are more important than others, so make sure to study them in detail!

Note: Remember that the final exam is *cumulative*. It contains 3 linear algebra questions and 4 differential equations questions. Also, remember that Grünbaum's coupled harmonic oscillators-question will be on the final for sure!

Suggestion: Start with the differential equations-part, since it's fresh in your mind. Once you're done with that, turn your attention to the linear algebra-part.

Differential equations

Suggestion: Even though the book spends a **LOT** of time with chapters 4 and 6, they are actually not important! When I took Math 54, I didn't even have questions from those two chapters. That's why, if I were you, I would just *briefly* review them, and focus my attention to chapters 9 and 10, which are **WAY** more important!

Chapter 4: Linear Second-order equations

- **Find the general solution to a second-order differential equation, possibly including complex roots, repeated roots, or initial conditions** (4.2.1, 4.2.11, 4.2.15, 4.3.1, 4.3.3)
- Determine if two functions are linearly independent or linearly dependent (4.2.27, 4.2.29)
- Solve equations using undetermined coefficients, but do not spend *too* much time on this (4.4.9, 4.4.11, 4.4.15, 4.4.17, 4.5.29, 4.9.3)
- Solve equations using variation of parameters (4.6.1, 4.6.3, 4.6.15)
- Find the equation motion of a simple harmonic oscillator (4.8.1, 4.8.11)

Chapter 6: Theory of higher-order linear differential equations

- **Find the largest interval on which a differential equation has a unique solution** (6.1.1, 6.1.3, 6.1.5)
- **Determine if a set of functions is linearly independent or linearly dependent** (6.1.9, 6.1.11, 6.1.13, 6.2.25)

Date: Tuesday, December 6th, 2011.

- **Find the general solution of a higher-order differential equation, possibly including initial conditions** (6.2.1, 6.2.5, 6.2.9, 6.2.15, 6.2.17, 6.2.19)

Chapter 9: Matrix methods for linear systems

- **Determine if a set of vector functions is linearly independent or linearly dependent** (9.4.15, 9.4.17)
- Determine if vector functions form a fundamental solution set for $\mathbf{x}' = A\mathbf{x}$ (9.4.19, 9.4.21)
- Use the fundamental matrix to find a solution to a given initial-value problem (9.4.27)
- **Find the general solution to $\mathbf{x}' = A\mathbf{x}$, possibly including complex eigenvalues** (9.5.13, 9.5.15, 9.5.19, 9.5.21, 9.5.31, 9.5.33, 9.6.1, 9.6.3)
- **Find the proper frequencies and proper modes of a coupled harmonic oscillator** (see 'Harmonic Oscillator'-Handout on my website)
- Solve inhomogeneous systems using undetermined coefficients (9.7.3, 9.7.5)
- Solve inhomogeneous systems using variation of parameters (9.7.11, 9.7.13, 9.7.15)

Chapter 10: Partial differential equations

- **Calculate the Fourier series of a function f on a given interval, and determine to which function that Fourier series converges** (10.3.9, 10.3.17, 10.3.11, 10.3.19, 10.3.13, 10.3.21)
- Calculate the Fourier cosine/sine series for a function f, and determine to which function that Fourier series converges to (10.4.5, 10.4.7, 10.4.9, 10.4.13, for the second part, you need to understand oddification and evenification, so see 10.4.1, 10.4.3)
- **Using separation of variables, solve the heat, wave, and Laplace equation, subject to various boundary/initial conditions** (10.5.1, 10.5.3, 10.5.5, 10.5.7, 10.5.11, 10.6.1, 10.6.3, 10.6.5, 10.6.13, 10.6.15, 10.7.1, 10.7.3, 10.7.5)

  Note: Do not waste your time reading sections 10.5, 10.6, 10.7, they contain tons of useless material! Just know how to solve the problems! Also, *don't* worry about inhomogeneous heat equations (10.5.9, 10.5.13), or about the Dirichlet problem on a disk (10.7.7)

Less important stuff

- Sketch trajectories of solutions to $\mathbf{x}' = A\mathbf{x}$ (9.6.17)
- Calculate e^{At}, by either diagonalizing A or using the Cayley-Hamilton theorem (9.8.1, 9.8.5, 9.8.7)
- Show that a given set is orthonormal, and calculate the generalized Fourier series for a given function f in terms of the orthonormal set (10.3.26, 10.3.27)

LINEAR ALGEBRA

Suggestion: Ignore chapters $1, 2, 3$, because you basically already know how to do them! Instead, focus on chapters $4, 5, 6, 7$

Chapter 4: Vector Spaces

- Determine whether a set is linear independent or dependent (4.3.3, 4.4.27)
- Find a basis and state the dimension of a vector space (4.5.1, 4.5.3, 4.5.5, 4.5.7, 4.5.9, 4.5.11)
- Given a matrix A, find a basis for $Nul(A)$, $Col(A)$, $Row(A)$, and also find $Rank(A)$ (4.2.3, 4.2.5, 4.3.9, 4.3.11, 4.5.13, 4.5.15, 4.5.17, 4.6.1, 4.6.3)
- Use the rank-nullity theorem to find $Rank(A)$ etc. (4.6.7, 4.6.9, 4.6.11, 4.6.13, 4.6.15)
- Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$ (4.7.7, 4.7.9)
- Use the change-of-coordinates matrix to find $[\mathbf{x}]_{\mathcal{C}}$ given $[\mathbf{x}]_{\mathcal{B}}$ (4.7.1, 4.7.3)

Chapter 5: Diagonalization

- **Find a diagonal matrix D and a matrix P such that $A = PDP^{-1}$, or say A is not diagonalizable** (5.2.9, 5.2.11, 5.2.13, 5.3.9, 5.3.11, 5.3.17)
- Show that a given matrix is not diagonalizable (5.4.11)
- **Find the matrix of a linear transformation** (5.4.1, 5.4.3, 5.4.9, 5.4.11, 5.4.17(b))

Chapter 6: Inner Products and Norms

- Determine if a set is orthogonal, or orthonormal (6.2.3, 6.2.19, 6.2.21)
- Find the orthogonal projection of $\mathbf{x}$ on a subspace W. Use this to write $\mathbf{x}$ as a sum of two orthogonal vectors, and to find the smallest distance between $\mathbf{x}$ and W (6.2.11, 6.3.3, 6.3.5, 6.3.1, 6.3.7, 6.2.15, 6.3.11)
- **Use the Gram-Schmidt process to produce an orthonormal basis of a subspace W spanned by some vectors** (6.4.1, 6.4.3, 6.4.5, 6.4.7, 6.4.9, 6.4.11)
- **Find the least-squares solution (and least-squares error) of an inconsistent system of equations** (6.5.1, 6.5.3, 6.5.7, 6.5.9, 6.5.11)
- Find inner products, lengths, and orthogonal projections of functions f and g using fancier inner products $\langle f, g \rangle$ (6.7.3, 6.7.5, 6.7.7, 6.7.9, 6.7.11, 6.7.22, 6.7.24)
- Show a given formula defines an inner product (6.7.13)
- Use the Gram-Schmidt process to find an orthonormal basis of **functions** (6.7.25, 6.7.26)
- Remember the Cauchy-Schwarz inequality (6.7.19, 6.7.20)

Chapter 7: Symmetric matrices and quadratic forms

- **Given a symmetric matrix A, find an diagonal matrix D and an orthogonal matrix P such that** $A = PDP^T$ (7.1.13, 7.1.15, 7.1.17, 7.1.21)
- Transform a quadratic form into one with no cross-product terms (7.2.9, 7.2.11)

TRUE/FALSE EXTRAVAGANZA

Do the following set of T/F questions: 1.5.24, 1.7.21, 1.7.22, 1.9.23, 1.9.24, 2.1.15, 2.8.21, 3.2.27, 4.1.24, 4.2.25, 4.3.21, 4.6.17, 4.7.11, 5.3.21, 6.3.21, 6.5.17 (check out the hints to HW 1-8 for answers)

CONCEPTS

Understand the following concepts:

- Pivots (1.2.23, 1.2.24, 1.2.25, 1.2.26, 1.5.29, 1.5.31)
- Span (1.3.22, 1.3.25, 1.4.17, 1.4.29, 1.4.34)
- Linear independence (1.7.33, 1.7.34, 1.7.35, 1.7.36)
- Invertible matrices (2.1.23, 2.1.24, 2.2.11, 2.2.19, 2.2.21)
- Implications of invertibility (2.3.11, 2.3.15, 2.3.17, 2.3.21, 2.3.24, 2.3.30)
- Subspace, Basis (2.8.1, 2.8.3, 2.8.5, 2.8.7, 2.8.17)
- Vector space, Subspace (4.1, 4.2)
- Basis, Dimension (4.3, 4.5)
- Coordinates of $\mathbf{x}$ with respect to $\mathcal{B}$ (4.4)
- $Nul(A)$, $Col(A)$, $Row(A)$, Rank (4.6)
- Rank-Nullity Theorem (4.6)
- Change of coordinates matrix (4.7)
- Eigenvalues, Eigenvectors, Characteristic polynomial (5.1 - 5.3)
- A is similar to B (5.2)
- Diagonalizable, Diagonalization Theorem (Theorem 5 in section 5.3)
- Matrix of a Linear transformation (5.4)
- Inner products, Norms, Orthogonal vectors, Orthogonal Matrix (6.1)
- Orthogonal projection (6.2, 6.3)
- Gram-Schmidt process (6.4)
- Least-squares (6.5)
- Inner product space (6.7)
- Cauchy-Schwarz inequality (6.7)
- Symmetric matrix (7.1)
- Quadratic form (7.2)

IMPORTANT THEOREMS

Know the following theorems (or write them on your cheat-sheet)

- Theorem 4 in section 1.4 (page 45)
- Theorem 12 in section 1.9 (pages 90-91)
- **The invertible matrix theorem** - Theorem 8 in section 2.3 and Theorem in section 2.9 and Theorem in section 4.6 (pages 131 and 165 and 255)
- The Spanning Set Theorem - Theorem 5b in section 4.3 (page 227)
- Theorem 12 in section 4.5 (page 247)
- **The Rank-Nullity Theorem** - Theorem 14 in section 4.6 (page 253)
- The Diagonalization Theorem - Theorem 5 in section 5.2 (page 288)
- Theorem 6 in section 5.2 (page 291)
- Theorem 5 in section 6.2 (page 325)
- Theorems 6 and 7 in section 6.2 (page 330)
- Theorem 10 in section 6.3 (page 339)
- The Cauchy-Schwarz inequality and the Triangle inequality - Theorems 16 and 17 in section 6.7 (page 372)
- Theorem 1 in section 7.1 (page 390)
- Theorem 2 in section 7.1 (page 391)
- Existence and uniqueness theorem: Theorem 1 in section 4.2 and Theorem 1 in section 6.1 (pages 457 and 538)
- Pointwise convergence of Fourier series: Theorem 2 in section 10.3 (page 653)